Lake Water Balance and Mass Balance

Surface water-groundwater interaction may have substantial influence on water levels and solute concentrations in lakes and wetlands.

Water and mass balance of a lake can be seen as an 'integrated' measurement of SW-GW exchange fluxes over the entire lake.

Objectives

1. **Understand the effects of SW-GW exchange on lake water and mass balance.**
2. **Estimate lake-scale average exchange rates from water balance simulation.**

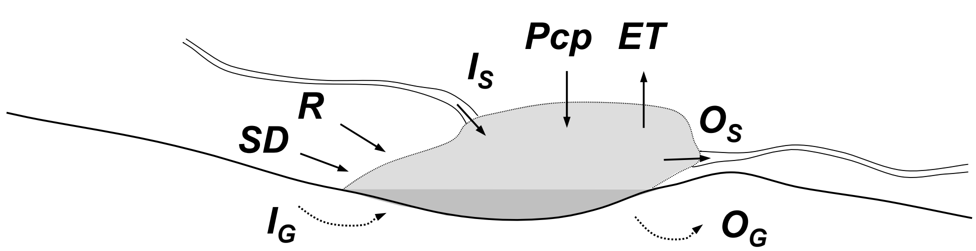
Textbook chapter

Rosenberry and Hayashi (2013. Assessing and measuring wetland hydrology, *In: Wetland techniques*, Springer, vol. 1, pp. 87-225).

Lake Water Balance Equation

$$\underset{\text{input}}{Q_{in}} - \underset{\text{output}}{Q_{out}} = \frac{dV}{dt} \cong A\frac{dh}{dt}$$

V: lake water volume (m^3)
dV/dt: rate of volume change (m^3 d^{-1})
A: lake water area (m^2)
h: water depth (m)



Input (m^3 d^{-1})

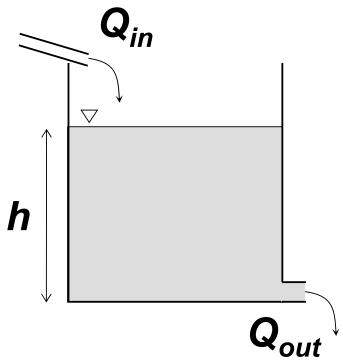
Pcp: precipitation
I_S: stream inflow
I_G: groundwater inflow
R: diffuse runoff

Output (m^3 d^{-1})

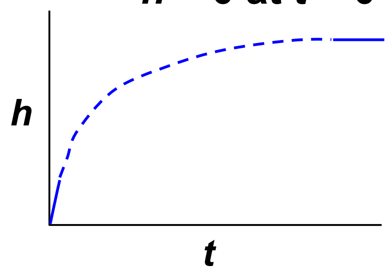
ET: evaporation & transpiration
O_S: stream outflow
O_G: groundwater outflow

SD: snow drift (in or out)

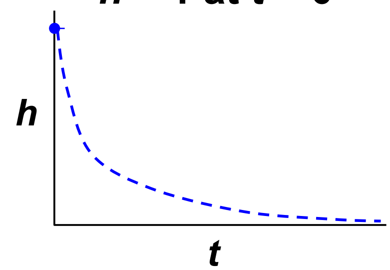
Simple 'Tank' Model of Water Balance



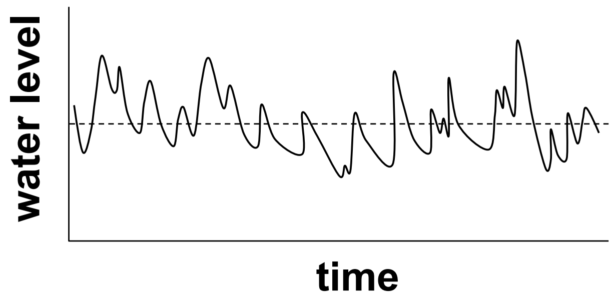
Case 1: Q_{in} = const.
$h = 0$ at $t = 0$



Case 2: $Q_{in} = 0$
$h = 1$ at $t = 0$

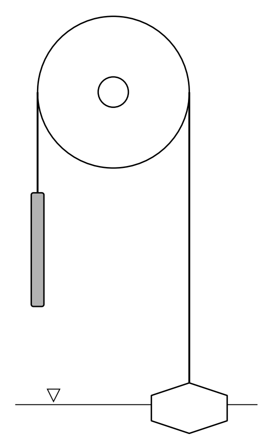


$Q_{out} \propto h \rightarrow$ **negative feedback**

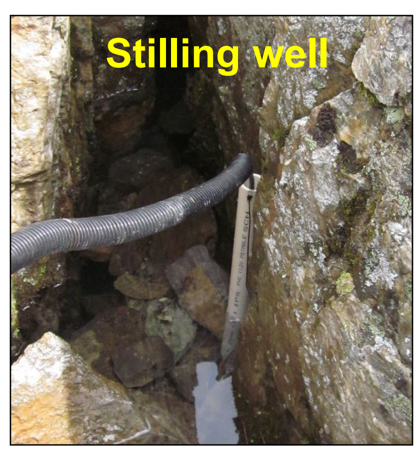


Natural systems are usually in a <u>steady state</u>, when averaged over a long time.

Float-pulley system



Pressure transducer



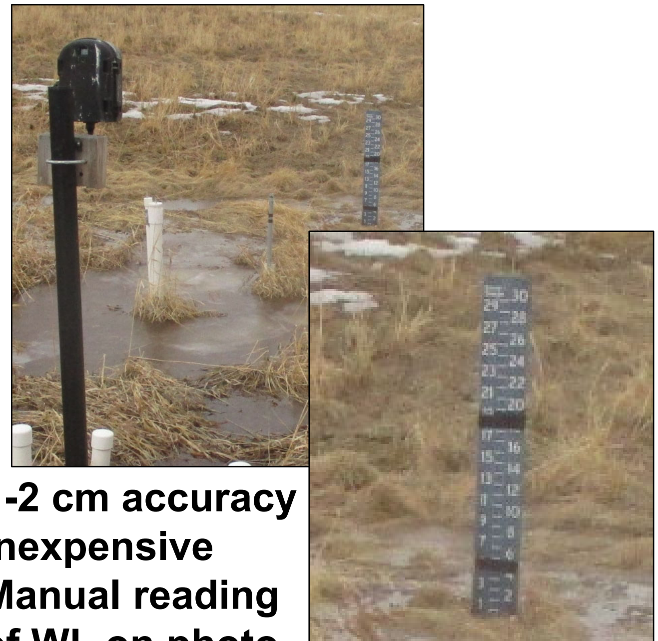
Rosenberry and Hayashi (2013. Assessing and measuring wetland hydrology. In: *Wetland techniques*)

Sensors for Cold-Season Measurements

Sonic distance sensor

- 2-3 mm accuracy
- Needs a datalogger

Time-lapse camera



- 1-2 cm accuracy
- Inexpensive
- Manual reading of WL on photo

Citizen Science Approach

CrowdHydrology project

- Citizens read the water level on a staff gauge.
- Send the data as SMS text.

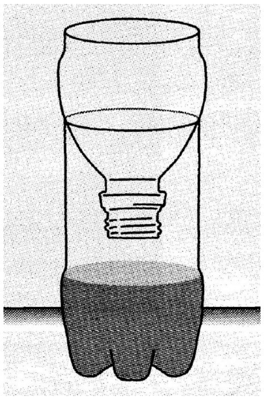
CrowdWater project

- Citizens take a photo of physical or 'virtual' staff gauge using a smart phone.
- Process the data on the smart phone app and send them in.

https://crowdwater.ch/en/

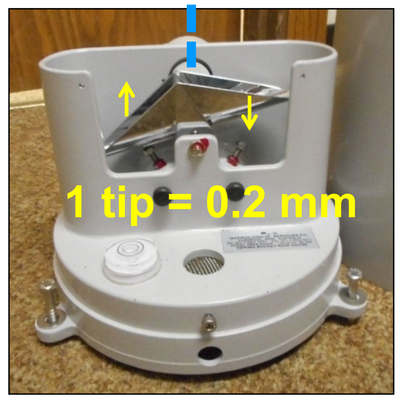
Precipitation Gauges

Non-recording rain gauge



Hendricks (2010. *Intro. to Physical Hydrol.*)

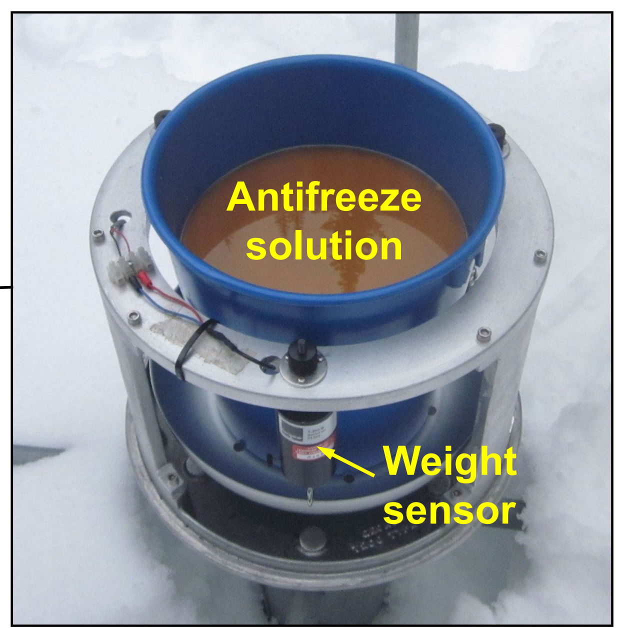
Tipping bucket rain gauge



- **Inexpensive**
- **Frequent attendance required**
- **No data for rainfall intensity**

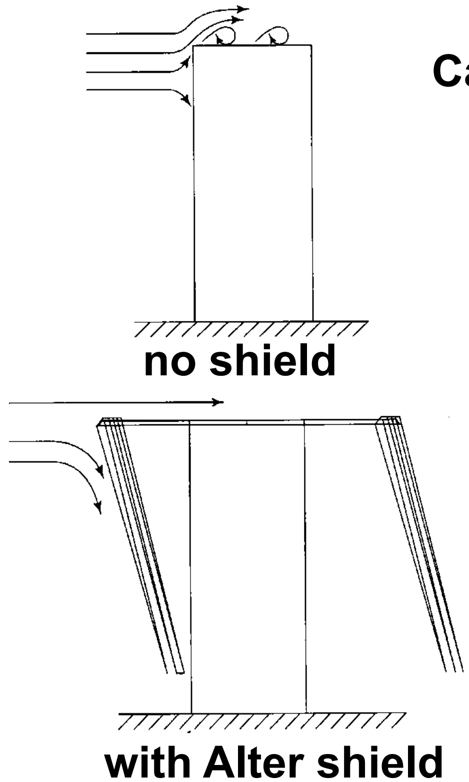
Can we measure snow with these?

Weighing Precipitation Gauge

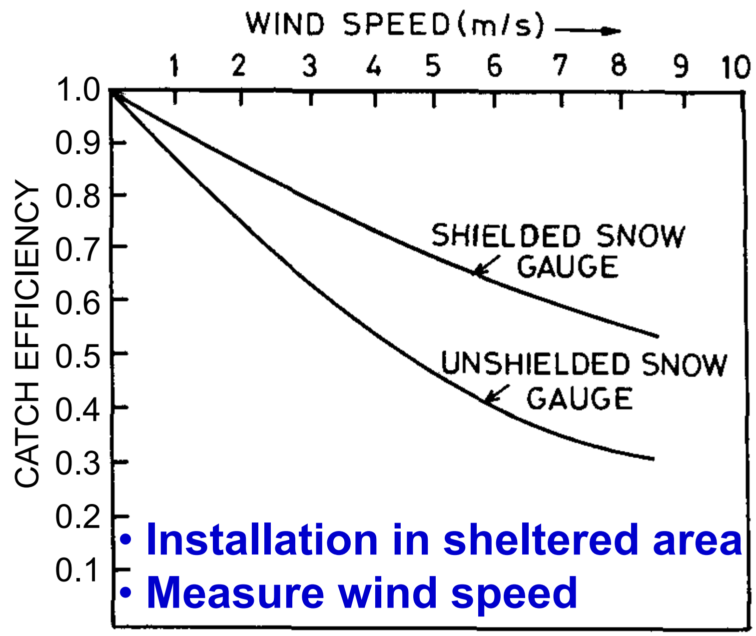


Wind-Induced Undercatch of Snow

$$\text{Catch efficiency} = \frac{\text{Measured precip.}}{\text{Actual precipitation}}$$



Dingman (2002, *Physical hydrology*)



- Installation in sheltered area
- Measure wind speed

Singh and Singh (2001, *Snow and glacier hydrology*)

Even shielded snow gauges require wind correction.

Archived climate data are often 'uncorrected', resulting in inaccuracy and inconsistency.

Example: A recent study by Meteorological Service of Canada (Mekis & Vincent. 2011. Atmosphere-Ocean 49: 163-177) showed substantial differences between corrected and uncorrected precipitation data.

e.g., Calgary annual pecipitation (1981-2010)
420 mm → 475 mm (13 % increase by correction)

Climate data interpretation for long-term trend analysis requires a special attention.

- **What type of gauge was used?**
- **Was correction made? How?**

e.g., WMO Standard: Kochendorfer et al. (2017. *Hydrol. Earth System Sci.*, 21, 3525-3542)

Snow Drift or Blowing Snow

Wind driven transport can move a large amount of snow, particularly on smooth, non-vegetated surfaces.

Snow drift also enhances the sublimation loss of snow.

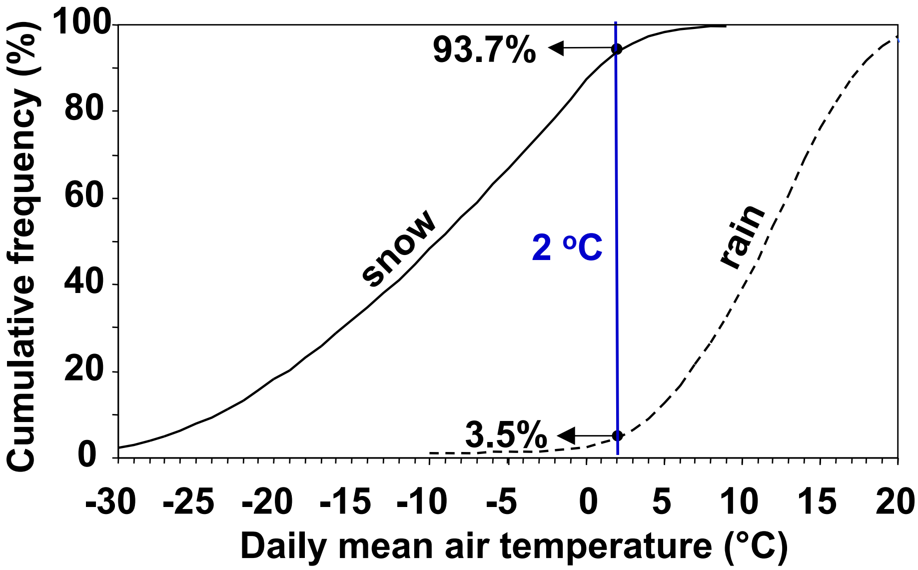
Do lakes lose or gain snow during blowing snow events?

What is the effect of tall stubble in farm fields?

Rain or Snow?

Automated meteorological stations record total amounts of precipitation, but not rain and snow individually.

Climate models calculate total precipitation and temperature. How can we separate rain and snow?

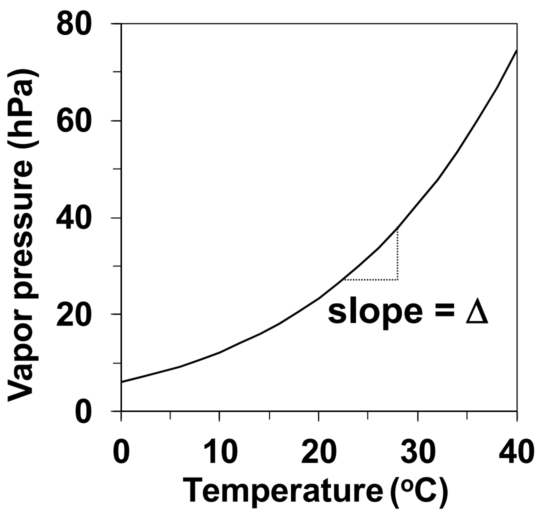


Data by Alberta Agriculture and Forestry

Background Information for Evaporation

What is relative humidity?

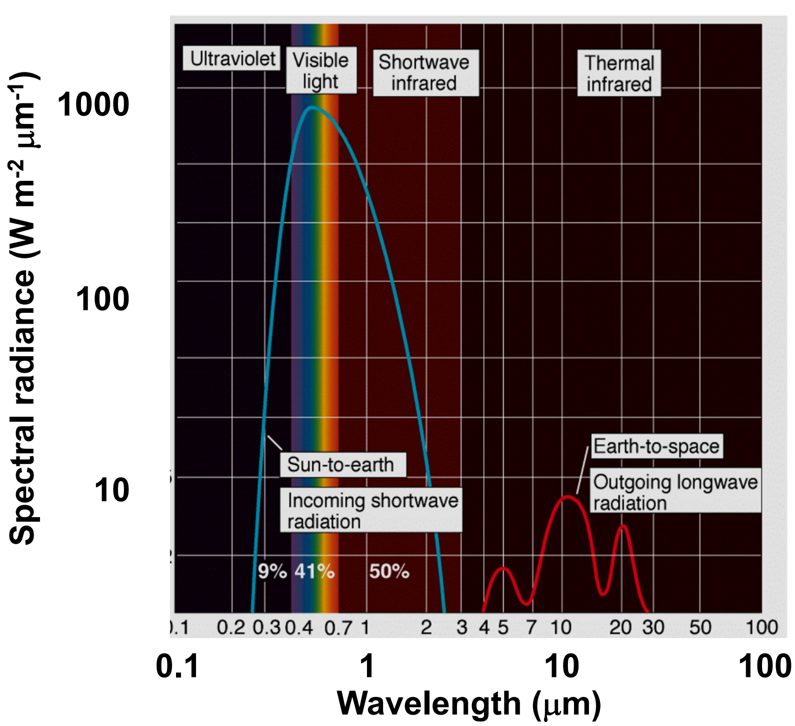
Amount of water vapour in the atmosphere is expressed as partial pressure. Maximum possible amount at a given temperature is called <u>saturation vapour pressure</u>.



The slope of the temperature-vapour pressure curve (Δ) has a special significance in the estimation of evaporation.

Radiation: Energy Source for Evaporation

Solar radiation has relatively <u>short</u> wavelengths, while the radiation from the earth has <u>long</u> wavelengths.

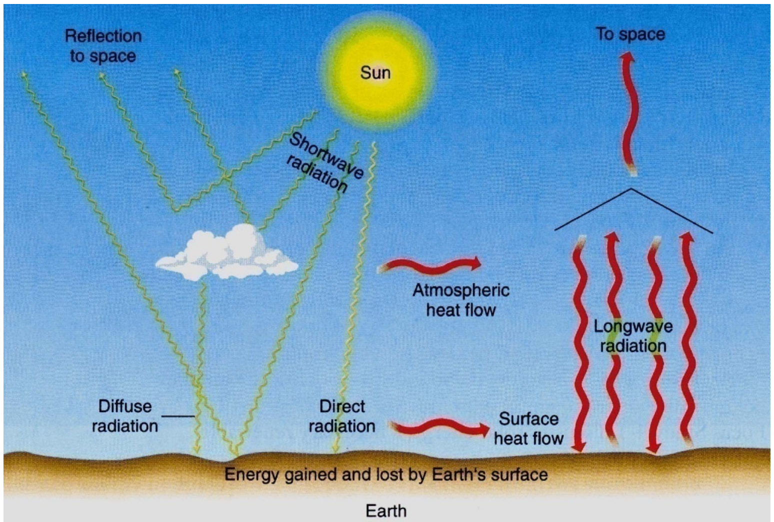


(Christopherson, 2000. *Geosystems*, Prentice-Hall)

Radiation Balance

Net radiation = incoming – outgoing radiation
= (incoming SW + LW) – (outgoing SW + LW)

Ratio of outgoing SW / incoming SW is called <u>albedo</u>.

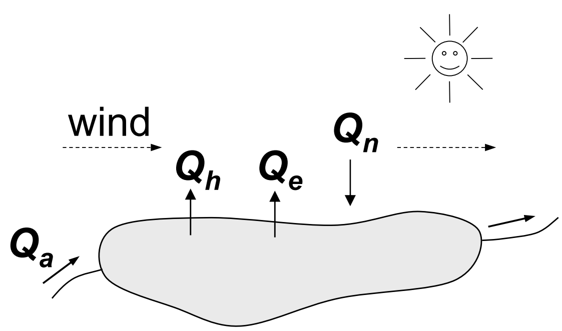


Christopherson (2000, Fig. 4-1)

Lake Energy Balance

$Q_n + Q_a - Q_h - Q_e \cong Q_w$ **(all terms in W m^{-2})**

- Q_n: **net radiation**
- Q_a: **net advection of energy by streams (and groundwater)**
- Q_h: **sensible heat flux**
- Q_e: **latent heat flux**
- Q_w: **rate of energy storage in lake water**



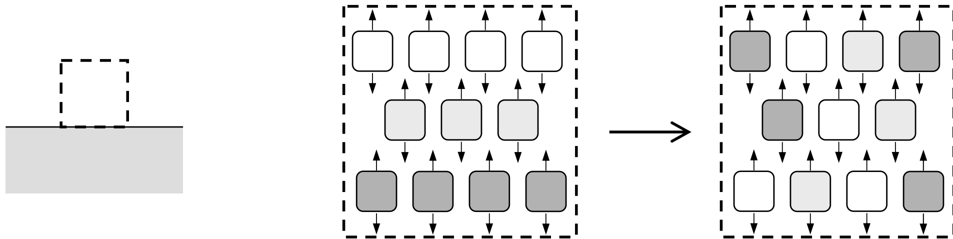
$Q_w \propto$ **volume × temp. change**

Evaporation rate, *E* (m s^{-1}) is proportional to latent heat flux.

$Q_e = E \times$ **density of water × latent heat of vaporization**

How is *E* affected by meteorological conditions?

Turbulent Flux of Latent Heat and Sensible Heat



Imagine a box over a lake. Each air 'parcel' within the box contains numerous molecules. Parcels near the water surface contain more water vapor than the ones far from the surface.

As the wind causes turbulent mixing within the box, random motion of the parcels lead to the net upward transfer of water vapor $\rightarrow$ Latent heat flux

Does the same principle apply to sensible heat flux?

What controls the magnitude of flux?

Bowen Ratio

Wind speed and temperature (or humidity) controls the flux.

$Q_e \propto f(u) \times (e_s - e_a)$

$Q_h \propto f(u) \times (T_s - T_a)$

u: horizontal wind speed (m s^{-1})

$f(u)$: wind function (m s^{-1}); e.g., $f(u) = a + bu$.

e_s: vapour pressure at the lake surface (hPa)

e_a: vapour pressure in the air above the lake (hPa)

T_s: temperature of the lake surface (°C)

T_a: air temperature above the lake (°C)

Same wind function for Q_e and Q_h. Why?

The wind function is complex, dependent on many factors (what are they?), but the ratio of Q_h to Q_e is relatively simple.

$\beta = Q_h / Q_e = \gamma (T_s - T_a) / (e_s - e_a)$ Bowen ratio

γ: psychrometric constant ($\cong$ 0.66 hPa °C^{-1} at sea level)

Estimation of Lake Evaporation

From the lake energy balance,

$$Q_h + Q_e \cong Q_n + Q_a - Q_w \quad \leftarrow \text{Available energy}$$

Using the Bowen ratio, $Q_h = \beta Q_e$

$$\therefore \; Q_e = (Q_n + Q_a - Q_w) / (1 + \beta)$$

Written in a different form, the Priestley-Taylor equation is:

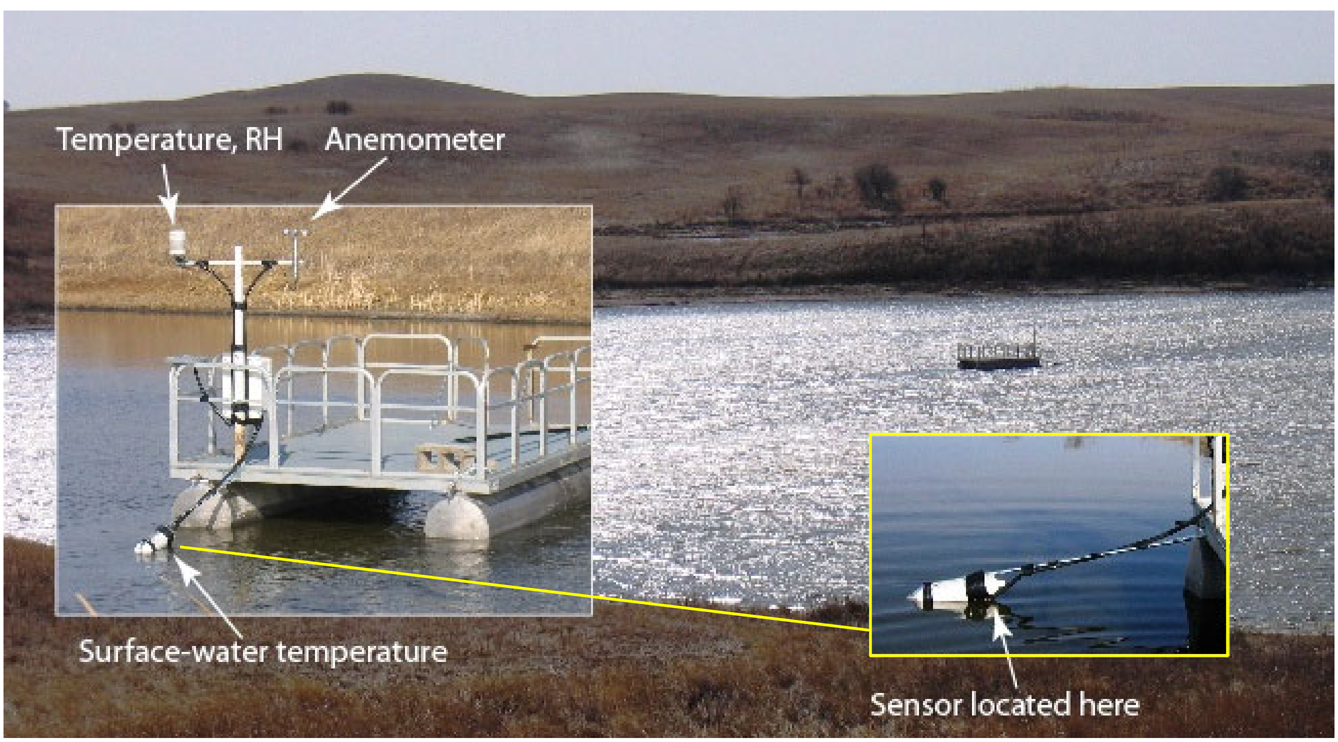
$$Q_e = (Q_n + Q_a - Q_w) \times \alpha \times \Delta / (\Delta + \gamma)$$

Δ: slope of vapour pressure-temperature curve

α: dimensionless constant

The equation with α = 1.26 has been shown to give reasonably accurate estimates of evaporation from shallow lakes and wetlands (Rosenberry et al., 2004, *Wetlands*, 24:483).

Measurement of meteorological variables



Rosenberry and Hayashi (2013. In: *Wetland techniques*)

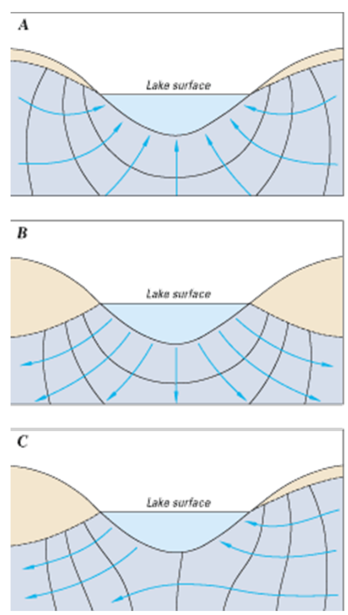
Langston et al. (2013. *Water Resour. Res.*, 49:5411-5426)

Groundwater Exchange with Lakes

Lakes are almost always connected to groundwater.

The amount and direction of groundwater exchange depends on topographic setting, geology, climate, and many other factors.

Water balance of some lakes are dominated by groundwater exchange, while other lakes are dominated by surface water inputs and outputs.

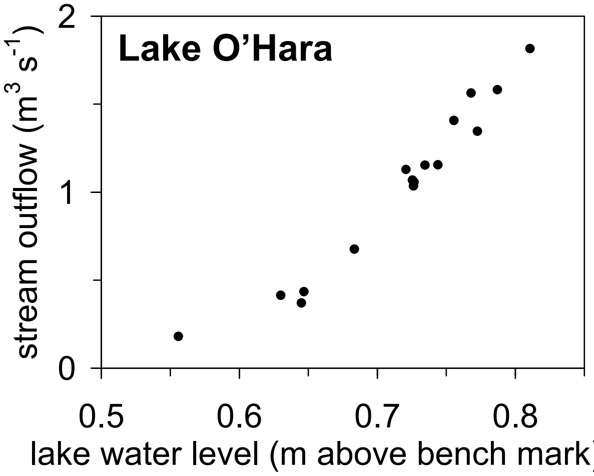


Winter et al. (1998. USGS Circular 1139)

Stream Inflow and Outflow

For lakes with inflow and outflow streams, accurate flow measurement is critical for lake water balance.

Outflow is controlled by lake water level → negative feedback

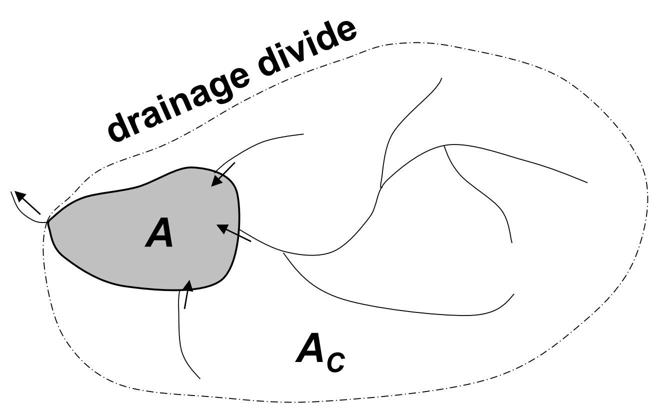


Runoff and Stream Inflow: Watershed Hydrology

Surface water input (m^3) = runoff (m) × A_c (m^2)

What controls runoff?

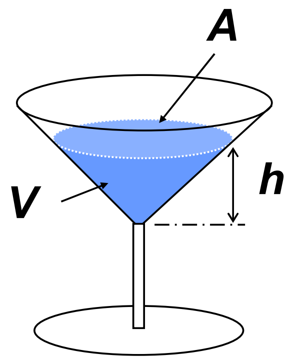
- **Climate**
- **Topography**
- **Soil thickness**
- **Geology**
- **Vegetation and landuse**
-

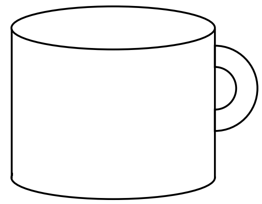


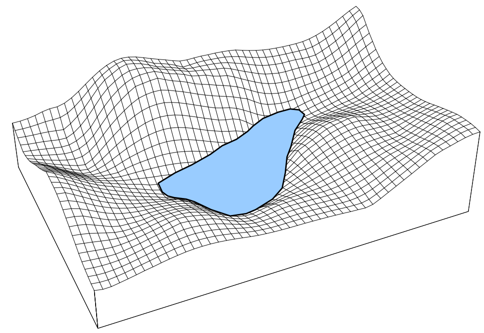
***A*: lake area A_C: catchment area**

Basin Morphology

Volume (*V*) - Area (*A*) – Depth (*h*) Relation



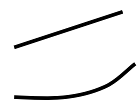




$A \propto h^2$	$A \propto h^0$	$A \propto h^{2/p}$
$V \propto h^3$	$V \propto h^1$	$V \propto h^{(1+2/p)}$

p **: parameter representing the slope 'profile'.**

- ***p*** **= 1 for straight slope**
- ***p*** **> 1 for concave slope**



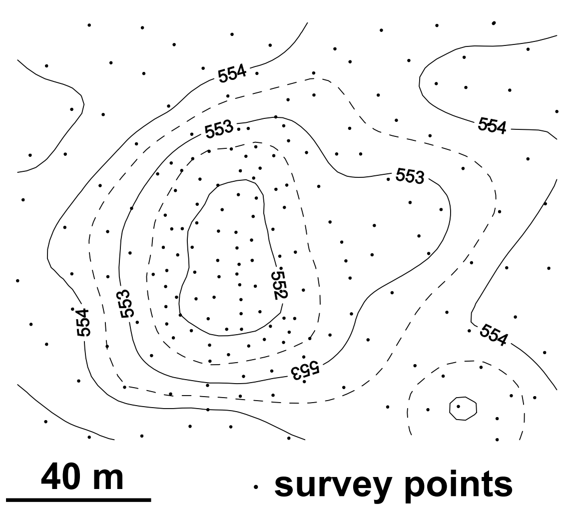
Hayashi and van der Kamp (2000, *J. Hydrol.*, 237: 74)

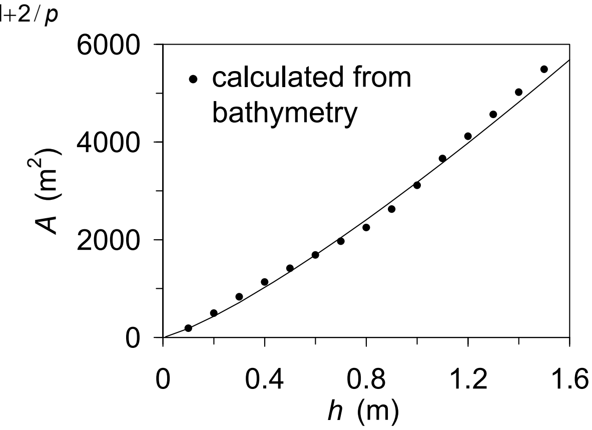
Volume (*V*) - Area (*A*) - Depth (*h*) Model

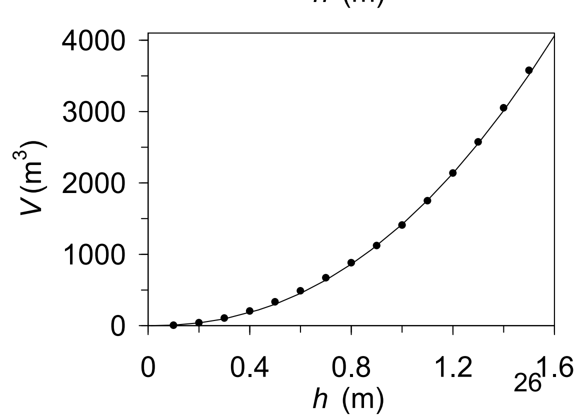
$$A = A_{\max}\left(\frac{h}{h_{\max}}\right)^{2/p} \qquad V = \frac{A_{\max}h_{\max}}{1+2/p}\left(\frac{h}{h_{\max}}\right)^{1+2/p}$$

A_{max}: maximum area

h_{max}: maximum depth







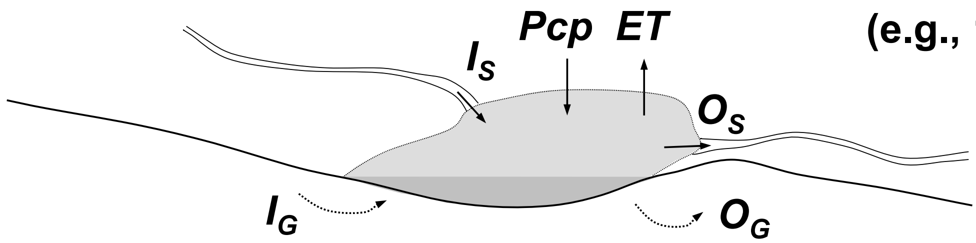
Simple Lake Water Balance Simulation

$$Q_{in} - Q_{out} = \frac{dV}{dt}$$

For lakes with negligible snow drift and diffuse runoff,

$$Pcp + I_S + I_G - ET - O_S - O_G = \Delta V/\Delta t$$

ΔV: volume change
Δt: time interval (e.g., 1 day)



***Pcp*, *ET*, and surface flows can be measured, but groundwater components are very difficult to measure. We will use the water balance equation to estimate net groundwater flow.**

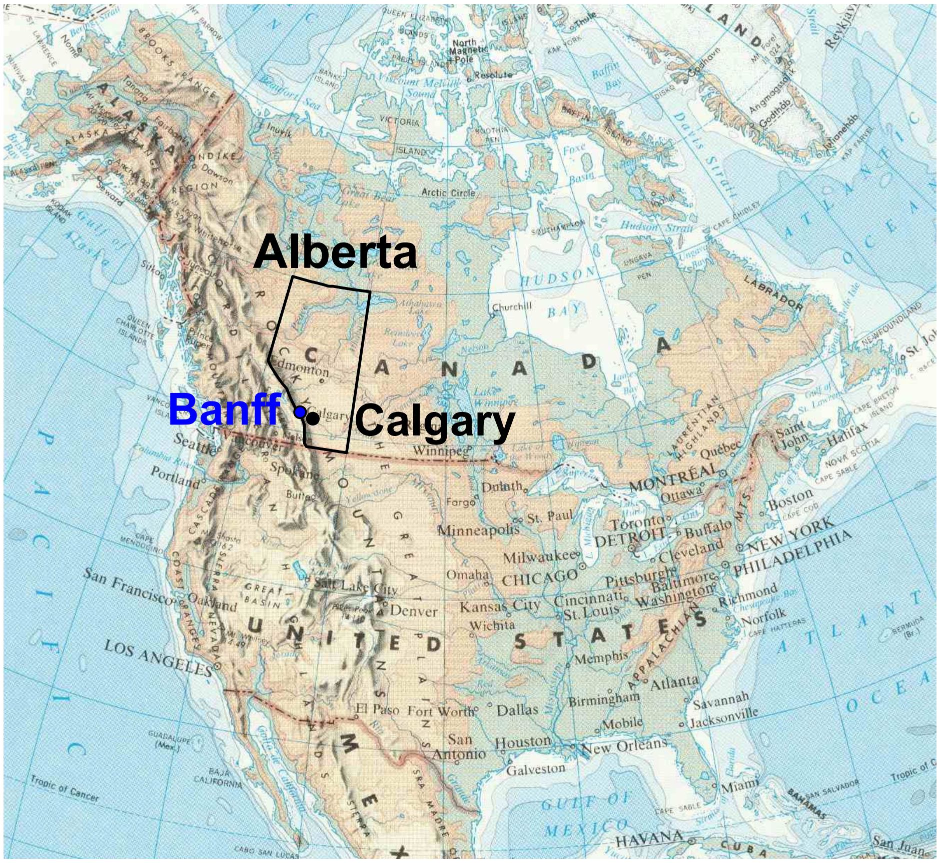
$$I_G - O_G = \Delta V/\Delta t - Pcp + ET - I_S + O_S$$

Simple Spreadsheet Exercise of Water Balance Simulation

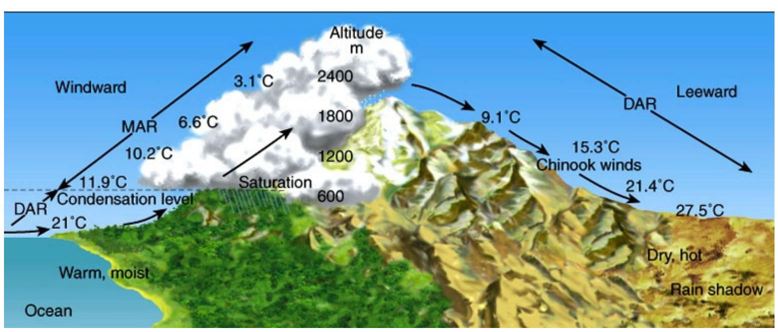
We will use Microsoft Excel to demonstrate a simple simulation of lake water balance.

Field data from a study site in Canada will be used as examples.

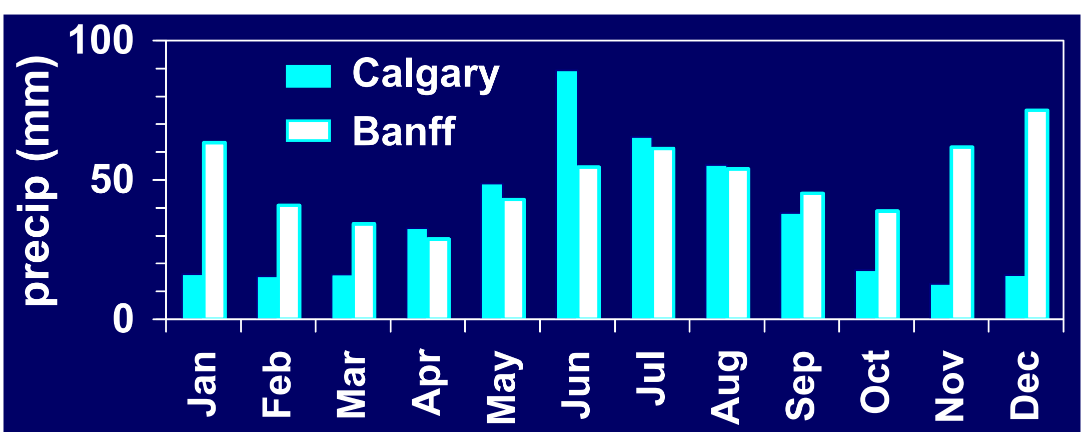
Alberta and Calgary



Mountains as 'Water Tower'



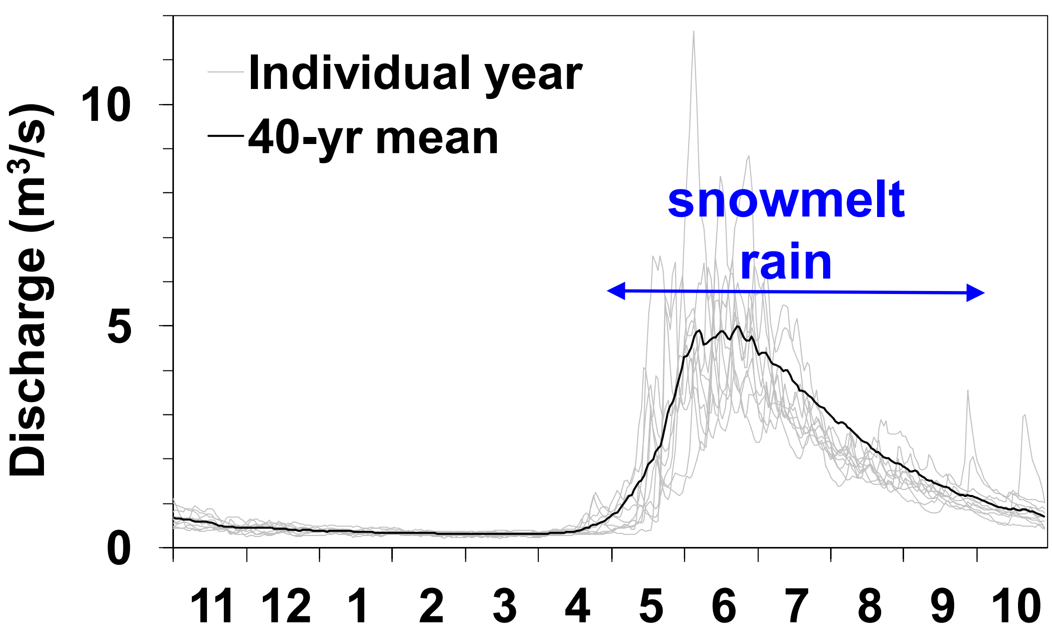
Christopherson (2000)



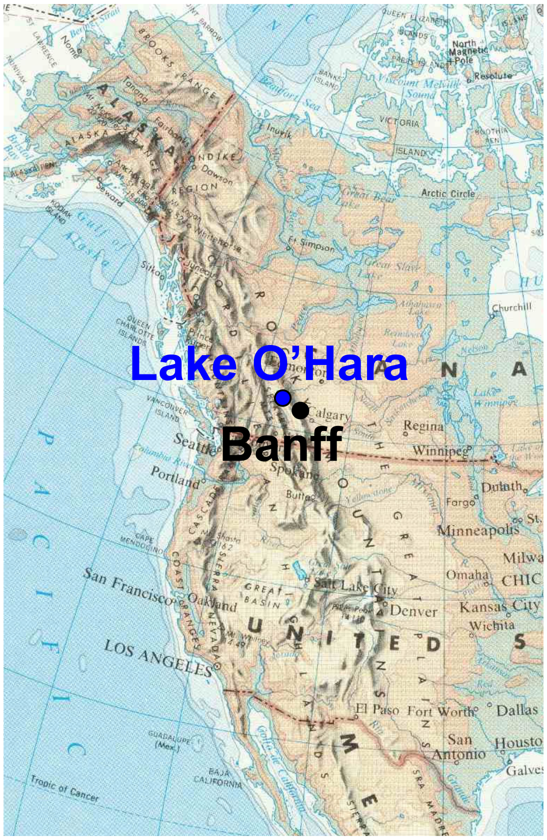
Mountain Rivers Provide Water Supply

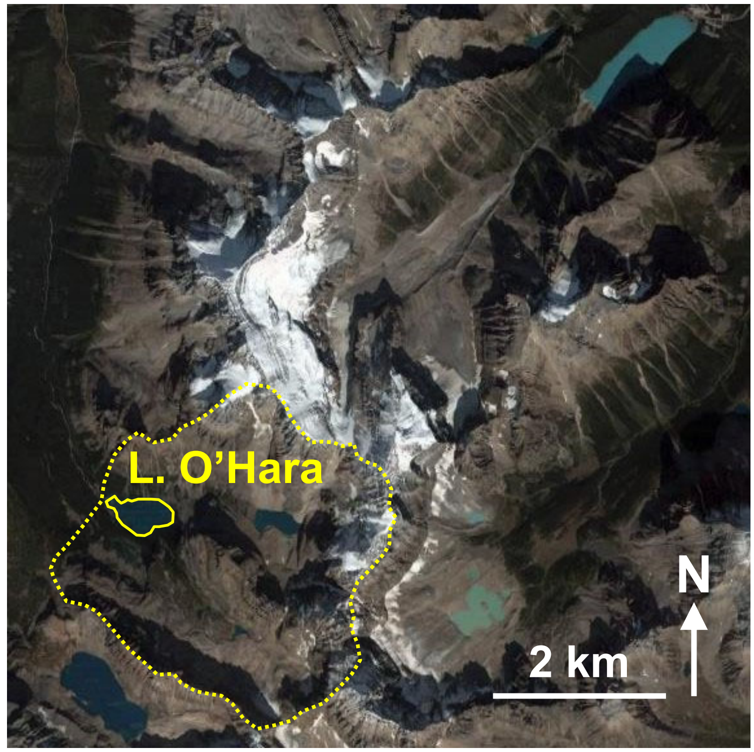
Measured River Flow, 2001-2010

Bow River at Banff (2200 km^2, unregulated)



Lake O'Hara Watershed (14 km^2)
Elevation: 2000-3500 m

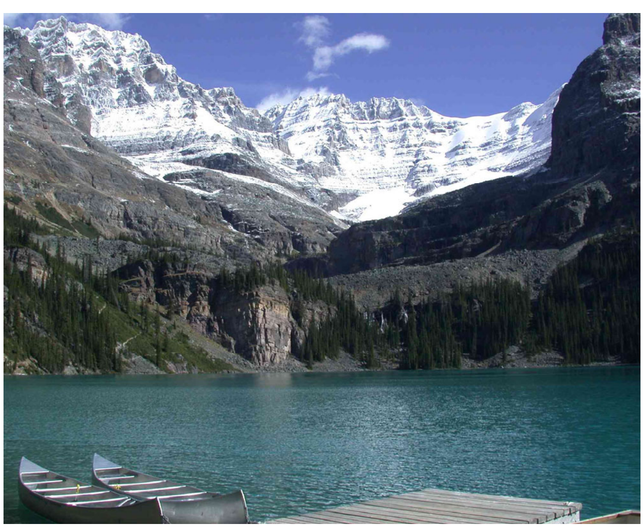




Lake O'Hara Hydrological Study

Issue: Climate change impacts on glaciers and water resources

How important is groundwater in alpine headwaters?



Lake O'Hara at 2000 m altitude

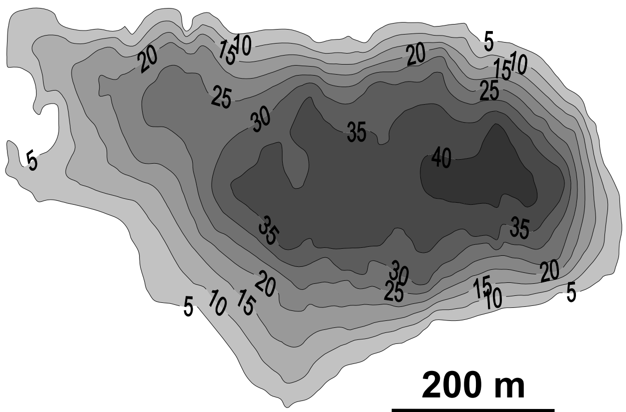
Opabin Glacier at 2500 m

Supplementary reading: Hood et al. (2006, *Geophys. Res. Lett.*, 33, L13405)

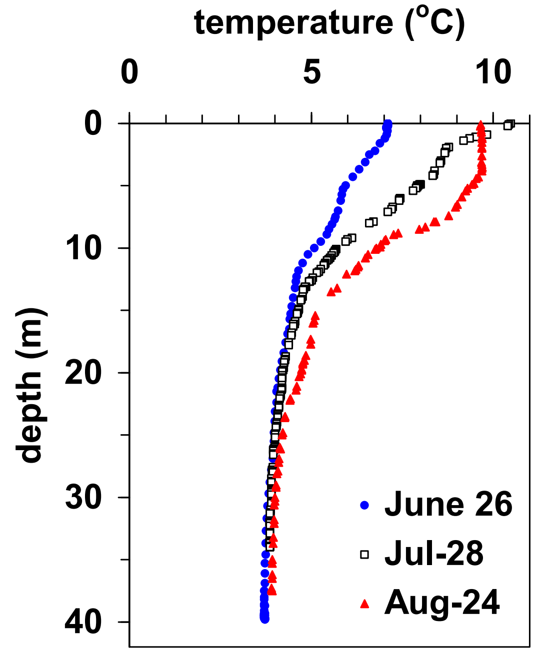
Lake O'Hara Characteristics

Frozen from November to May.

Area = 0.26 km^2
Max depth = 42 m



Depth contour = 5 m



Evaporation Estimate by Priestley-Taylor Eqn.

(72 + 0 – 23) 0.57 / (0.57+0.52)

$$Q_e = (Q_n + Q_a - Q_w) \times \alpha \times \Delta / (\Delta + \gamma)$$

1.26

Q_n: net radiation, measured (photo)
Q_a: advection by streams, ignored (expected to be minor)
Q_w: energy storage in lake, from temperature profiles

For June 3-15, 2005, Q_n = 72 W m^{-2} Q_w = 23 W m^{-2}

Avg. temp = 4.1 ^{o}C $\rightarrow \Delta$ = 0.57 hPa K^{-1}

At 2000 m elev., γ = 0.52 hPa K^{-1}

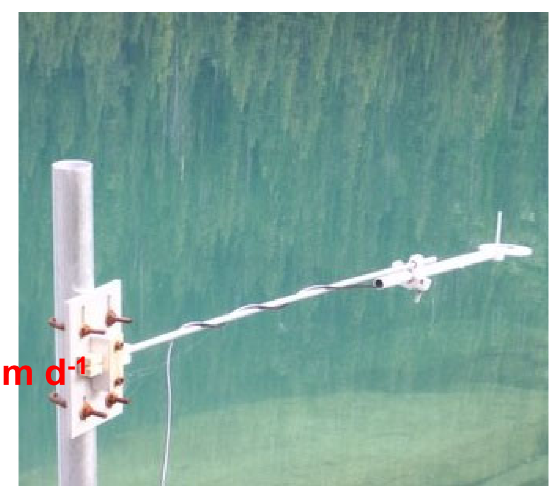
Assume α = 1.26

Q_e = 32.3 W m^{-2}

Latent heat (L_v) = 2.49 × 10^6 J kg^{-1}

Density (ρ_w) = 1000 kg m^{-3}

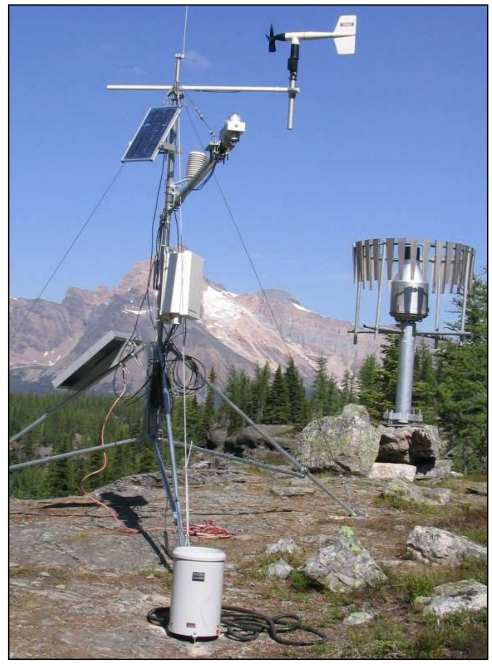
$$E = Q_e / (L_v \rho_w) = \frac{32.3 \text{ J s}^{-1} \text{ m}^{-2}}{2.49\times10^9 \text{ J m}^{-3}} = 1.1 \text{ mm d}^{-1}$$



Precipitation and Stream Flow Measurements

Estimated uncertainty in flow measurements ≅ 10 %

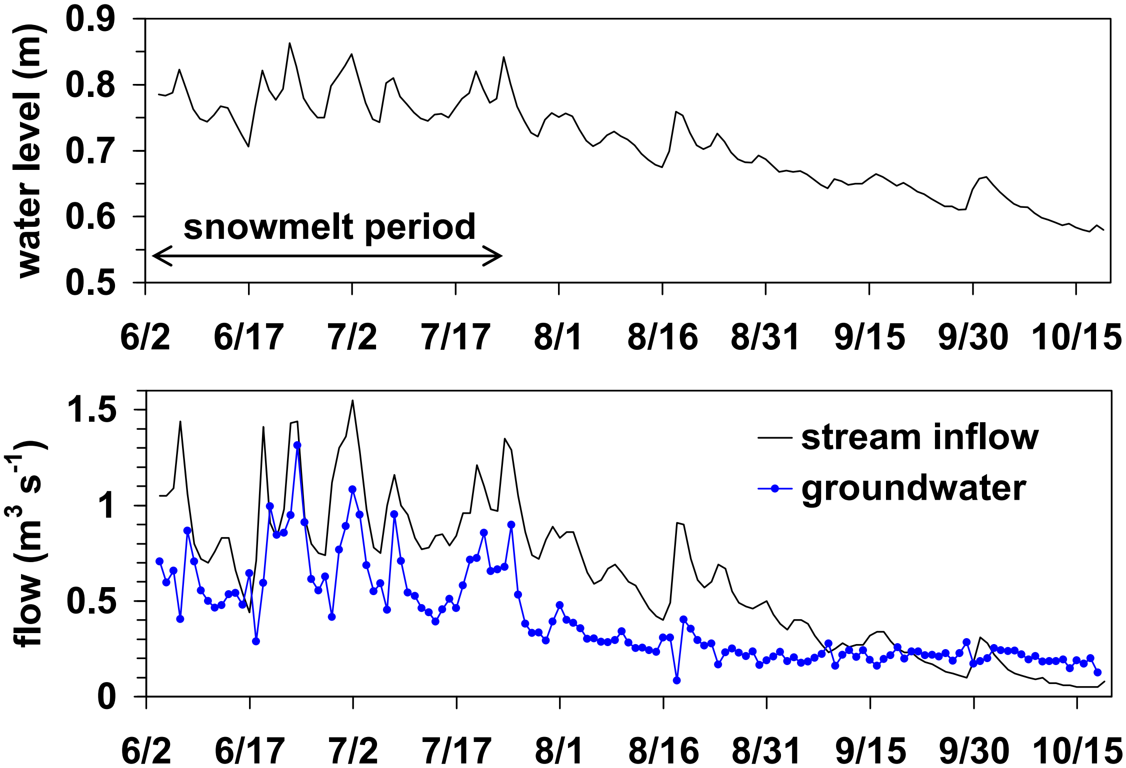
Jaime Hood gauging a stream



Tipping bucket rain gauge

See Exercise 4 for step-by-step instructions.

Lake Water Level (w.r.t. Bench Mark) and Flux



Lake Solute Mass Balance Equation

Solute mass balance is similar to water balance. Each term is multiplied by the concentration. For example;

C_P (kg m^{-3}) × Pcp (m^3 d^{-1}) = mass flux (kg d^{-1}) in precip.

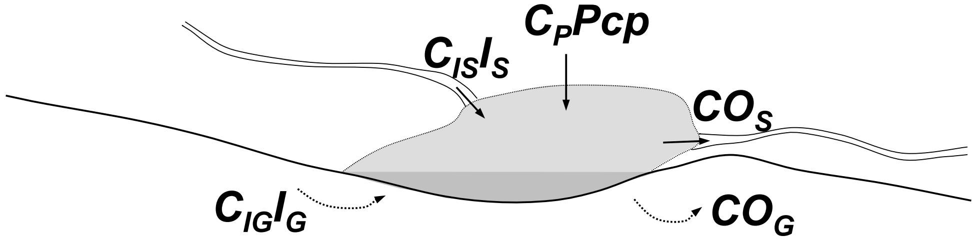
C (kg m^{-3}) × V (m^3) = total mass (kg) in the lake.

Mass balance equation is:

$$[C_P Pcp + C_{IS} I_S + C_{IG} I_G - C(O_S + O_G) + R_{XN}]\,\Delta t = \Delta(CV)$$

C: Concentration in lake (kg m^{-3})

R_{XN}: Reaction rate (kg d^{-1})



Solute Mass Balance

$$[C_P Pcp + C_{IS} I_S + C_{IG} I_G - C(O_S + O_G) + R_{XN}]\,\Delta t = \Delta(CV)$$

The concentration of outflow terms is equal to *C*. What is the underlying assumption?

Why is *ET* not in the equation?

Reaction term (R_{XN}) represents all other processes. What are those?

- **Dissolution/precipitation of minerals**
- **Biological production (e.g. CO_2) and uptake (e.g. N and P)**
- **Atmospheric exchange**
- **Diffusive exchange with the sediment**

Combining water (WB) and mass balance (MB)

WB: $Pcp + I_S + I_G - ET - O_S - O_G = \Delta V/\Delta t$

MB: $C_P Pcp + C_{IS} I_S + C_{IG} I_G - C(O_S + O_G) + R_{XN} = \Delta(CV)/\Delta t$

Two equations can be solved simultaneously for I_G and O_G.

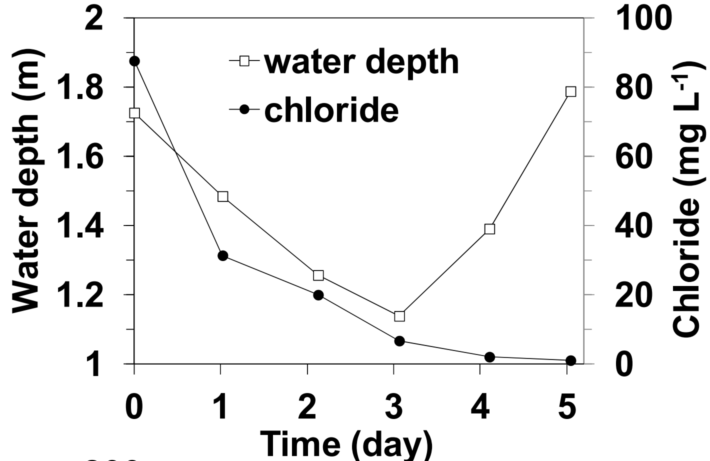
Example: GW flow through a pond with no surface flow.
Conservative tracer: e.g., chloride.

WB: $Pcp + I_G - ET - O_G = \Delta V/\Delta t$

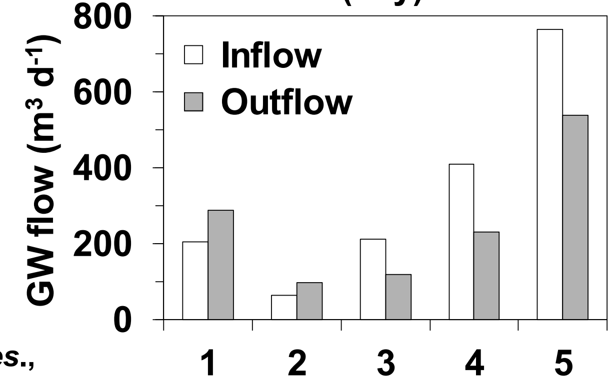
MB: $C_{IG} I_G - C O_G = \Delta(CV)/\Delta t$

***Pcp*, *ET*, *V*, *C* can be easily measured or estimated. If we have a good estimate of C_{IG}, we can determine I_G and O_G on a daily time step.**

Chloride Tracer Experiment in a Proglacial Pond

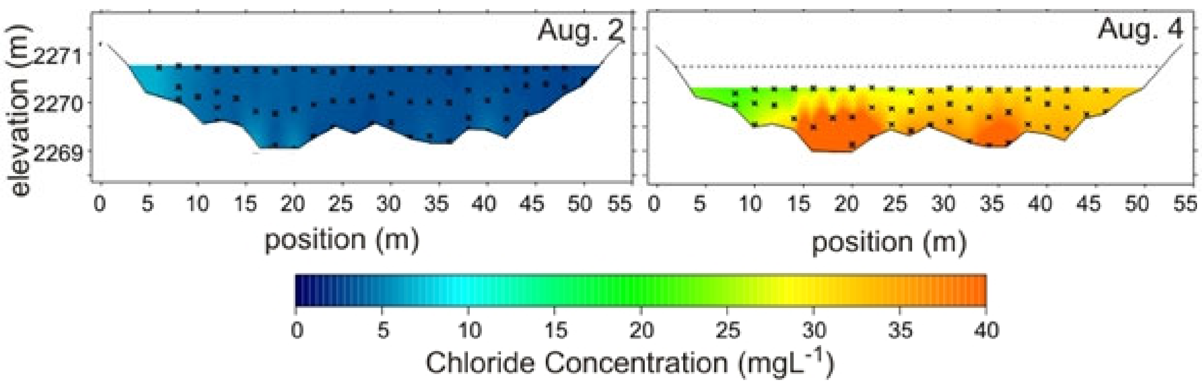


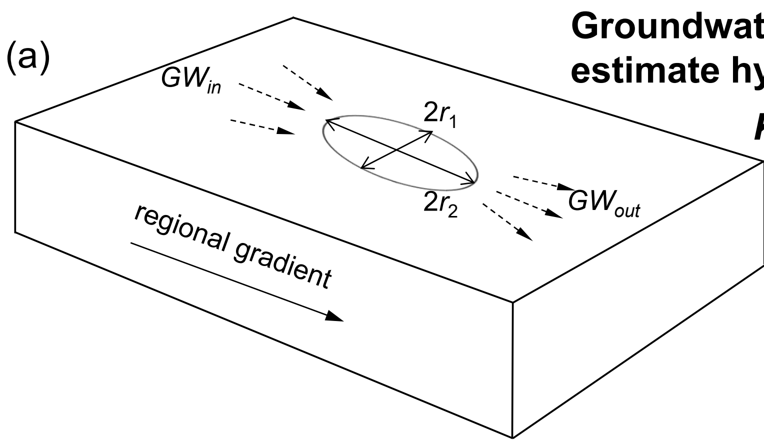
- **Tracer was released.**
- **Average concentration was determined daily from spatially distributed measurements.**



Langston et al. (2013. *Water Resour. Res.*, 49:5411-5426)

Distribution of Cl concentration

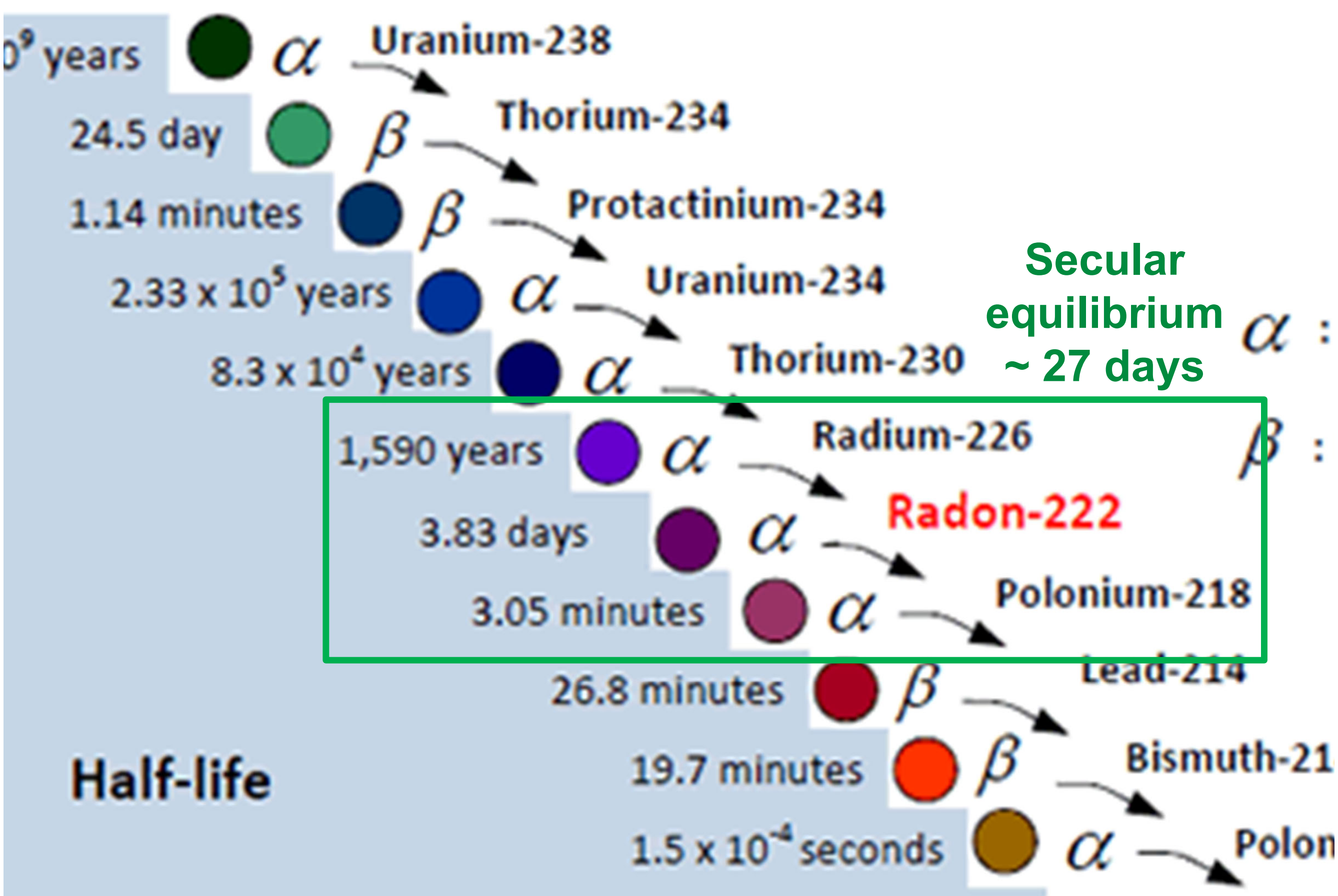




Groundwater flow model to estimate hydraulic conductivity

$K \approx 0.3\text{-}3 \times 10^{-3}$ m s^{-1}

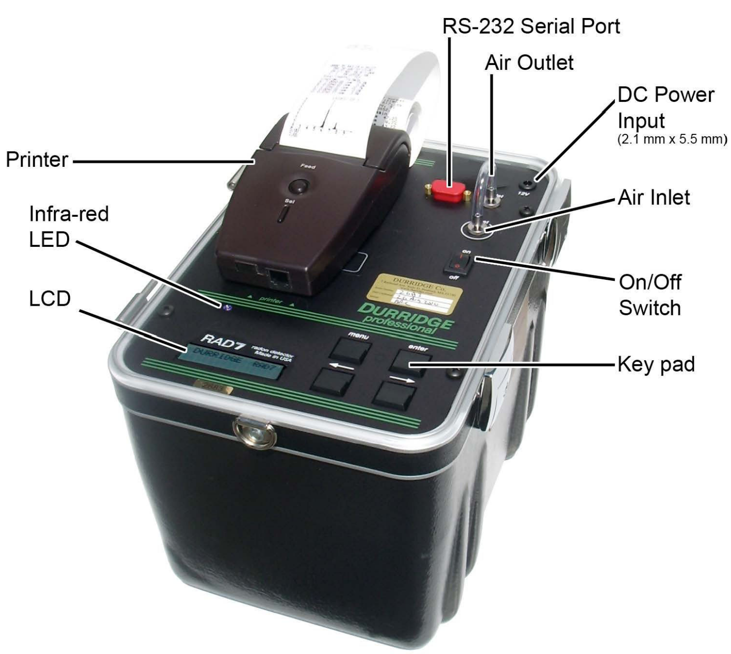
Quick Introduction to Radon methods



By measuring the activity of dissolved ^{222}R, it <u>may be</u> possible to:

- **Detect local discharge of groundwater**
- **Differentiate the pathways of groundwater**
- **Qualitatively infer the residence time of groundwater**
- **Infer the gas-exchange history of groundwater in karst systems.**
- **????**

Durridge RAD7 device



Radon extractor